

5.5

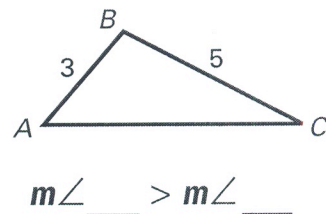
Inequalities in One Triangle

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- Goals**
- Use triangle measurements to decide which side is longest or which angle is largest.
 - Use the Triangle Inequality.

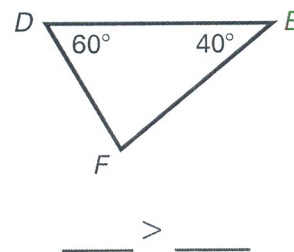
THEOREM 5.10

If one side of a triangle is longer than another side, then the angle opposite the longer side is _____ than the angle opposite the shorter side.



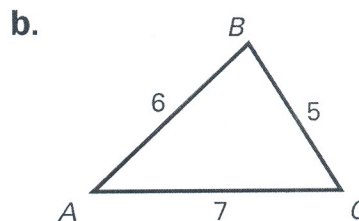
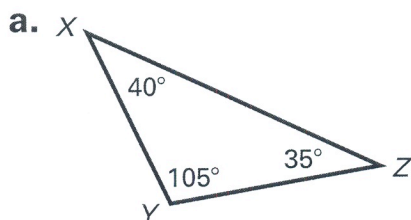
THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is _____ than the side opposite the smaller angle.



Example 1 Writing Measurements in Order from Least to Greatest

Write the measures of the triangles in order from least to greatest.



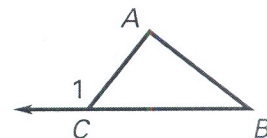
Solution

a. $m\angle \underline{\hspace{1cm}} < m\angle \underline{\hspace{1cm}} < m\angle \underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}} < \underline{\hspace{1cm}} < \underline{\hspace{1cm}}$

b. $\underline{\hspace{1cm}} < \underline{\hspace{1cm}} < \underline{\hspace{1cm}}$
 $m\angle \underline{\hspace{1cm}} < m\angle \underline{\hspace{1cm}} < m\angle \underline{\hspace{1cm}}$

THEOREM 5.12: EXTERIOR ANGLE INEQUALITY

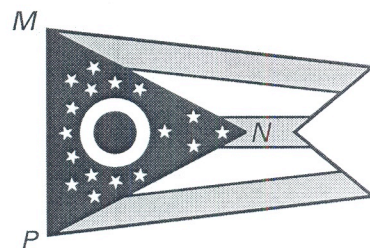
The measure of an exterior angle of a triangle is greater than the measure of either of the two nonadjacent interior angles.



$$m\angle 1 > m\angle \underline{\hspace{1cm}} \text{ and } m\angle 1 > m\angle \underline{\hspace{1cm}}$$

Example 2 Using Theorem 5.10

State Flags The state flag of Ohio is shown at the right. In the flag, $\overline{MN} \cong \overline{PN}$ and $MP < MN$. What can you conclude about the angle measures in $\triangle MNP$?



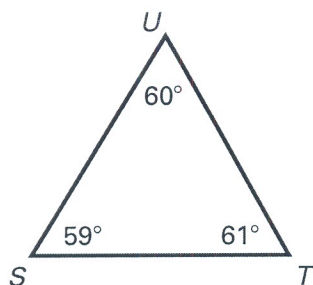
Solution

- Because $\overline{MN} \cong \overline{PN}$, $\triangle MNP$ is _____. So, $\angle \underline{\hspace{1cm}} \cong \angle \underline{\hspace{1cm}}$. Therefore, $m\angle \underline{\hspace{1cm}} = m\angle \underline{\hspace{1cm}}$.
- By Theorem 5.10, because $MP < MN$, $m\angle \underline{\hspace{1cm}} < m\angle \underline{\hspace{1cm}}$.
- Because $\overline{MN} \cong \overline{PN}$, $MN = PN$. So, by substitution, $\underline{\hspace{1cm}} < PN$. By Theorem 5.10, $m\angle \underline{\hspace{1cm}} < m\angle \underline{\hspace{1cm}}$.
- In addition, you can conclude that $m\angle M \underline{\hspace{1cm}} 60^\circ$, $m\angle N \underline{\hspace{1cm}} 60^\circ$, and $m\angle P \underline{\hspace{1cm}} 60^\circ$.

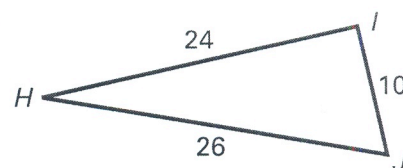
The sum of the angle measures in a triangle is 180° . In $\triangle MNP$, use logical reasoning to decide whether an angle measure is less than 60° or greater than 60° .

✓ **Checkpoint** Write the measures of the triangle in order from least to greatest.

1.



2.



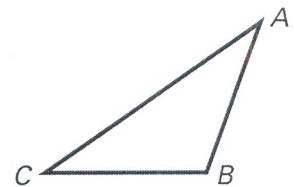
THEOREM 5.13: TRIANGLE INEQUALITY

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} > AC$$

$$AC + \underline{\hspace{1cm}} > \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} + AC > \underline{\hspace{1cm}}$$



Example 3 Finding Possible Side Lengths

A triangle has one side of 12 inches and another side of 20 inches. Describe the possible lengths of the third side.

Solution

Let x represent the length of the third side. Using the Triangle Inequality, you can write and solve inequalities.

$$x + \underline{\hspace{1cm}} > \underline{\hspace{1cm}}$$

$$x > \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} > x$$

$$\underline{\hspace{1cm}} > x$$

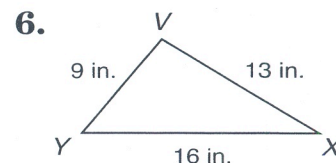
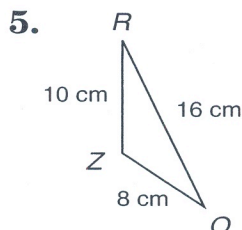
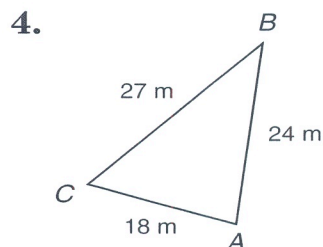
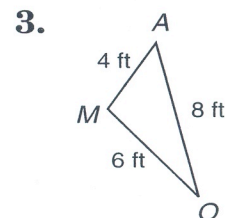
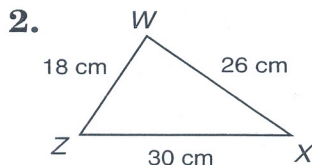
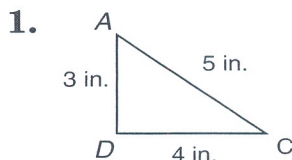
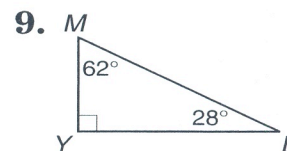
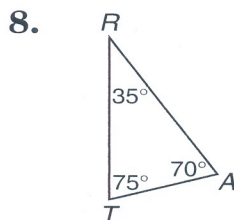
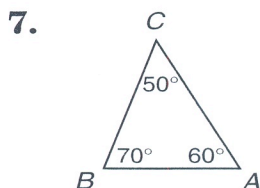
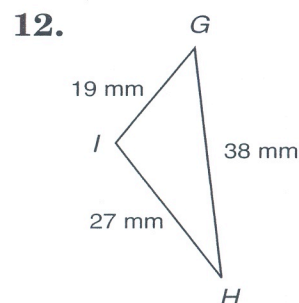
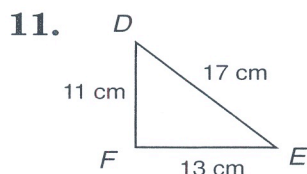
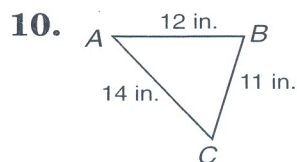
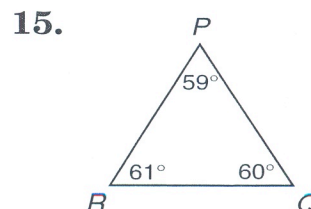
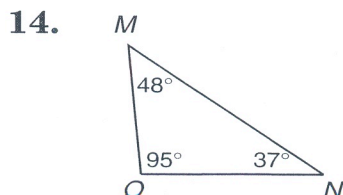
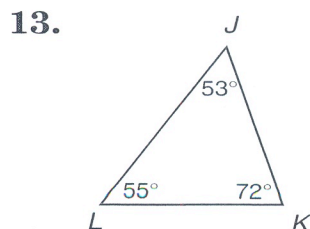
Answer The length of the third side must be greater than $\underline{\hspace{1cm}}$ inches and less than $\underline{\hspace{1cm}}$ inches.

✓ **Checkpoint** Decide if it is possible to construct a triangle having the given side lengths. If it is not possible, explain.

3. 13 mm, 25 mm, 14 mm

4. 9 in., 17 in., 8 in.

5. A triangle has one side of 8 millimeters and another side of 11 millimeters. Describe the possible lengths of the third side.

Skills Practice***Inequalities Within a Triangle*****List the angles in order from least to greatest measure.****List the sides in order from least to greatest measure.****Identify the angle with the greatest measure.****Identify the side with the greatest measure.**

Skills Practice

120

Triangle Inequality Theorem

Determine if the three numbers can be measures of the sides of a triangle. Write yes or no. Explain.

1. 6, 7, 8

2. 1, 1, 2

3. 2, 4, 6

4. 5, 8, 10

5. 10, 20, 30

6. 3, 4, 5

7. 3, 5, 7

8. 6, 12, 24

9. 1, 7, 10

10. 10, 15, 26

11. 8, 12, 19

12. 4, 7, 10

Find the range of possible measures for the third side of a triangle with the measures given for two sides.

13. 7, 13

14. 20, 25

15. 1, 5

16. 32, 38

17. 50, 70

18. 8, 20

19. 55, 10

20. 2, 10

21. 60, 70

22. 45, 70

23. 9, 19

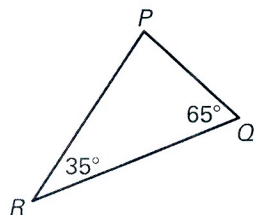
24. 100, 120

Practice A

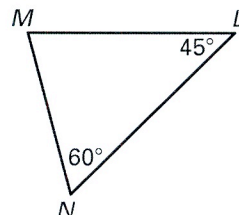
For use with pages 295–301

Name the shortest and longest sides of the triangle.

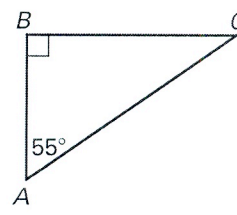
1.



2.

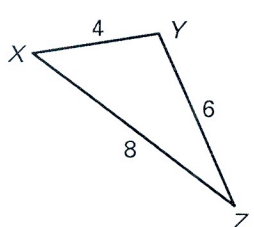


3.

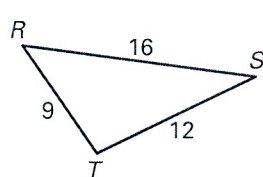


Name the smallest and largest angles of the triangle.

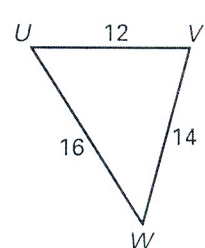
4.



5.

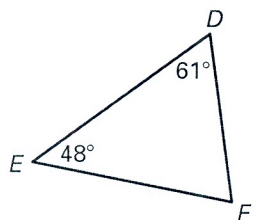


6.

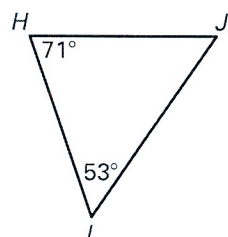


List the sides in order from shortest to longest.

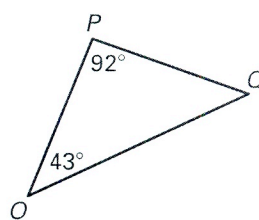
7.



8.

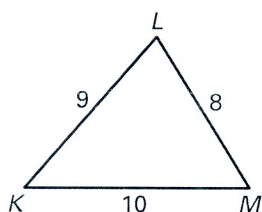


9.

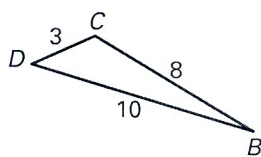


List the angles in order from smallest to largest.

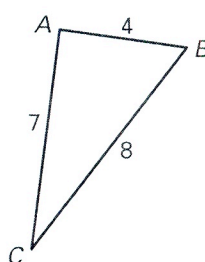
10.



11.



12.



In Exercises 13–15, you are given a 12-inch piece of wire. You want to bend the wire to form a triangle so that the length of each side is a whole number.

13. Sketch two possible isosceles triangles and label each side length.
14. Sketch a possible scalene triangle.
15. List two combinations of segment lengths that will not produce triangles.